Spin-orbit interaction and orbital effects in low dimensional semiconducting and superconducting nanostructures

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4 Presentation of scientific achievements forming the basis for habilitation procedure

As a scientific achievement within the meaning of Art. 16, par. 2 of the Act of 14 March 2003 “On Academic Degrees and Academic Title and on Degrees and Title in Art” I present a series of 11 related publications (in chronological order):


under a common title:

“Spin-orbit interaction and orbital effects in low dimensional semiconducting and superconducting nanostructures.”.
4.1 Introduction

Interaction of the electron spin with an external magnetic and electric field underlies the operation of many spintronics devices and is relevant in creation of topological states of matter. In my research, for several years I have been investigating the role of spin-orbit interaction and orbital effects in low-dimensional semiconducting and superconducting structures, in the context of applying these effects to the construction of spintronics devices and their impact on topological properties of semiconductor nanowires.

The interaction of electron spin with electromagnetic field in the vacuum is described by the relativistic Dirac equation which can be reduced to the Pauli equation by the Foldy-Wouthuysen transformation \[1\]

\[
\left[\frac{p^2}{2m_0} - \frac{e}{2m_0c^2} \mathbf{B} \cdot \mathbf{L} + \frac{e^2}{2m_0c^2} \mathbf{A}^2 + V + \frac{eh}{2m_0} \mathbf{B} \cdot \mathbf{B} + \frac{h}{4m_0^2c^2} \mathbf{p} \cdot (\mathbf{\sigma} \times \nabla V) \right. \\
- \frac{h^2}{8m_0^2c^2} \nabla \cdot \nabla V - \frac{p^4}{8m_0^2c^2} - \frac{ehp^2}{4m_0^2c^2} \mathbf{\sigma} \cdot \mathbf{B} - \frac{(ehB)^2}{8m_0^2c^2} \right] \psi = E\psi, \tag{1}
\]

where \(p\) is the momentum operator, \(m_0\) is the free electron mass, \(c\) is the light velocity, \(\mathbf{L} = \mathbf{r} \times \mathbf{p}\) is the orbital momentum, \(\mathbf{B}\) is the magnetic field, \(\mathbf{A}\) is the vector potential \((\nabla \cdot \mathbf{A} = 0)\), \(\mathbf{\sigma} = (\sigma_x, \sigma_y, \sigma_z)\) is the vector of Pauli matrices and \(V\) is the electrostatic potential.

Mathematically, the equation (1) is an expansion of the Dirac equation up to order \((v/c)^2\). The second term in this equation is related to the orbital effects and describes the interaction of the magnetic field with the orbital magnetic momentum \(\mu_L = \frac{e}{2m_0c} \mathbf{L}\). In Eq. (1) the spin-orbit (SO) interaction is described by

\[
H_{SO} = \frac{h}{4m_0^2c^2} \mathbf{p} \cdot (\mathbf{\sigma} \times \nabla V). \tag{2}
\]

The remaining terms are respectively: kinetic term, diamagnetic term, electrostatic potential, Zeeman spin splitting, Darwin term and higher-order corrections to both the kinetic energy and the Zeeman term. While the orbital effects appear as a result of the interaction of the orbital momentum with the magnetic field and are a non-relativistic effect, the spin-orbit interaction is a relativistic effect in which part of the electric field, in the rest frame of the electron, interacts with it like a magnetic field. In other words, if the electron moves in a vacuum in the external magnetic \((\mathbf{B})\) and \((\mathbf{F})\) electric field, measured in rest frame of the laboratory, the appropriate Lorentz transformation to the rest frame of the electron leads to the resultant field magnetic \(B_{eff} = B + B_{SO}\) being the sum of the external magnetic field and the effective magnetic field related to the spin-orbit interaction.

In semiconductor nanostructures, the electric field may arise from a symmetry breaking that is either intrinsic, i.e., related to the crystallographic structure - the SO interaction related to this kind of asymmetry is called Dresselhaus SO coupling \([2]\), or induced by the overall asymmetry of the confinement potential due to an electrostatic field from external gates - the so-called Rashba SO coupling \([3]\). Typically, SO coupling is the combination of both components, but zinc-blende nanostructures grown along \([111]\) direction posses inversion symmetry, and the Dresselhaus contribution vanishes \([1, 4]\). The presence of the strong Rashba coupling with negligible Dresselhaus component is one of the most important condition in the realization of many spintronics nanodevices, in which the key issue is the control of spin by the use of the external electric field.

One of the pioneer spintronics device is the spin field effect transistor proposed in 1990 by Datta and Das \([5]\). In this device, the Rashba SO coupling is used to control the electron spin in a semiconductor conduction channel (in the original idea, in the form of a two-dimensional electron gas) located between two ferromagnetic contacts. The spin of electron flowing through the conduction channel rotates around the effective SO field whereas the distance needed for a single spin rotation depends on the strength of the SO interaction. The channel resistance, which depends on the spin state at the output, can be controlled by the external potential applied to
the gate. My research on electronic transport in spintronics devices was mainly focused around the concept of the spin transistor.

Although the idea of spin transistor was proposed nearly 30 years ago, it is still waiting for the experimental realization. Recent experiments show that the application of the original concept proposed by Datta and Das leads to a low signal level at the output and low ratio in the ‘on’ and ‘off’ state of the transistor [6, 7], which is far away from the level required in modern electronics. The reason for that is a low efficiency of the spin injection in semiconductor materials [8] and the spin relaxation in the conduction channel [9], mainly due to Dyakonov-Perel [10] and Elliot-Yafet [11] mechanism. For these reasons, current studies on the spin transistor are conducted in two main directions. The first one is to eliminate all physical obstacles resulting in a low value of signal at the output. Therefore, it is direction to improve the original idea of the transistor. The second direction, more innovative, is focused on the experimental realization of a completely new architecture, different from that proposed in 1990 [5]. The first part of my research, the results of which I present as the scientific achievement, focused around both of these directions. In each of them the most important role was played by the spin-orbit interaction and the orbital effects. The second part of my achievement concerns the influence of the SO interaction and the orbital effects on Majorana states in semiconductor/superconductor hybrid systems. Below I will briefly present my contribution to these interesting fields of knowledge together with the context and motivation to undertake these studies. A detailed description of the papers presented as the scientific achievement can be found in the further part of this report.

In the original idea of spin transistor, the spin polarized electrons are injected into the conduction channel from ferromagnetic contacts. However, the difference of resistance between the ferromagnetic contact and the paramagnetic conduction channel (semiconductor) is so large that the maximum efficiency of spin injection through the ferromagnetic/semiconductor interface reaches only 70% at room temperature [12]. A simple theoretical model, which neglects the spin relaxation in the conduction channel, shows that the spin polarization of current at the level of 70% is not sufficient for applications in real electronic circuits as the signal ratio in the ‘on’ and ‘off’ state of transistor is too low, $G_{on}/G_{off} = (1 + P_S P_D)/(1 − P_S P_D) = 3$ [4], where $P_S(P_D) = 0.7$ is the efficiency of spin injection (detection) in the ferromagnetic source (drain). The same simple model shows that the ratio $G_{on}/G_{off}$ at the level required by modern electronics (at least $10^5$) is possible for the spin polarization of current $P = 99.999\%$. Such a high spin polarization can not be achieved by using ferromagnetic contacts. Therefore, the original concept of the spin transistor should be modified by using appropriate spin filters based on semiconductor materials. The use of semiconductor spin filters allows to eliminate the problem of resistance mismatch and obtain the required signal level in the ‘on’ and ‘off’ state. In this respect, I developed two models of spin filters based on semiconductors, which were presented in Refs. [H2, H7]. The first one is based on a semiconductor Y-shaped structure with the quantum point contact (QPC) located in one of the branches. In this model, the spin separation effect results from combined the Zeeman effect in QPC and the orbital effects which guarantee the spatial separation of the spin-polarized components of the current. The second proposed model is based on the inter-subband spin-orbit interaction in the coupled quantum wires with the quantum point contact.

Our theoretical model of the QPC-based spin filter in the coupled quantum wires [H7] is part of the current trend to produce an effective spin filter and a spin transistor solely based on semiconductor materials. Although the first experimental papers demonstrating the spin filtering effect in QPC appeared a few years ago, [13, 14, 15, 16, 17, 18, 19, 20, 21], an undoubted success of these systems was their application to the construction of a spin transistor solely based on semiconductor materials [22, 23]. It turns out that the use of the QPC-based spin filters instead of the ferromagnetic contacts leads to the signal ratio $G_{on}/G_{off}$ three orders of magnitude greater than that obtained by the use of ferromagnets [6]. Our concept of the spin filter [H7] is an extension of the idea proposed in Ref. [21] to the two coupled quantum wires. In the proposed model, the spin polarization is much higher than the value obtained in the QPC systems based
on a single quantum wire [17, 18, 21]. This allows to believe that the efficiency of spin transistor based on these filters will be much higher. In this respect, the structure of conduction channel, which should correspond to the structure of spin filter, also seems to be important. Therefore, it should has the form of the coupled quantum wires. In such systems, except the intra-band SO interaction, the inter-subband SO coupling becomes of high importance [24]. In my paper [H6] I analyzed the influence of the intra- and inter-subband SO coupling on the operation of spin transistor with the conduction channel based on the coupled quantum wires. I showed that the rapid change of the intra-subband SO coupling constants around zero gate voltage is favorable for applications in the spin-transistor architecture in which the transition between ‘on’ and ‘off’ states should be realized in the voltage range as narrow as possible.

Although recent papers show that the use of semiconductor spin filters can significantly improve the efficiency of the spin transistor, the problem of spin relaxation in the conduction channel still remains to be solved. For this reason, in the latest experiments the conduction channel is embedded in 1D nanowires [6, 25], in which the spin relaxation processes are strongly suppressed by the size effect [26, 27]. Due to the growing interest in spin transistors based on semiconductor nanowires, I performed the computer simulations of spin dependent electronic transport in the spin transistor based on InAs nanowire, in which the transistor action is induced by the Rashba spin-orbit coupling generated by the gate [H1]. The results of my calculations were compared with the experiment performed by the Japanese group of prof. J. Yoh [25] giving good agreement with the experimental data.

The key parameter in the fabrication of spin transistors based on 1D nanowires, which determines both their operation and the spin relaxation in the conduction channel, is the Rashba spin-orbit coupling constant [28, 29, 30, 31, 32]. The calculations of this parameter are currently one of my research directions. For this purpose I have established a cooperation with a group of theoreticians from University of Modena who specialize in the calculations of electronic structure in core-shell nanowires. In cooperation with them, I used the k·p approximation to determine the Rashba spin-orbit coupling constant in homogeneous semiconductor nanowires with hexagonal symmetry. Our results for InSb nanowires agree quantitatively with the experiment [31], in which the authors measured extremely large spin-orbit splitting 0.25 – 1 meV which corresponds to the Rashba coupling constant $\alpha = 50 – 100$ meV nm. Our further calculations concerned the Rashba SO interaction in core-shell nanowires. In Ref. [H11] we showed that in core-shell nanowires the interface contribution to the SO coupling can be much greater than the electrostatic component arising from the gate potentials. Our results qualitatively agree with the recent experiment by Furthmeier et al. in Ref. [33], where the enhancement of the SO coupling was measured in GaAs/AlGaAs core-shell nanowires.

The second direction of research on the spin-transistor is related to new concepts of the transistor architecture, different from that proposed in 1990 by Datta and Das. One such concept was presented by Betthausen et. al. in Ref. [34]. In this architecture, the spin control is realized by the combined homogeneous and helical magnetic fields. The latter is generated by ferromagnetic stripes located above the conduction channel. The spin state of electrons flowing through the channel is protected against a possible decay by keeping the transport in the adiabatic regime. The transistor action is driven by the diabatic Landau–Zener transitions [35] induced by the appropriate tuning of the homogeneous magnetic field. For these suitably chosen conditions, the backscattering of spin-polarized electrons appears which gives rise to the large increase of the resistance; i.e., the transistor goes over into the ‘off’ state. Motivated by this experiment, in the paper [H4] we studied the impact of non-adiabaticity on the electronic transport in the new architecture of the spin transistor. Our studies showed that in the non-adiabatic regime, the high resistance state can appear for several different values of the magnetic field. This phenomenon was explained as resulting from the resonant Landau-Zener transitions.

Although the new spin transistor architecture [34] seems to be free from physical limitations related to the original concept [5], such as the spin relaxation, the main drawback of the new solution is the transistor action induced by the external magnetic field. In response to this limi-
tation we proposed a new model of spin transistor based on a helical magnetic field, in which the transistor action is driven by the electric field [H5]. In the proposed model, the Landau-Zener transition is induced by the Rashba spin-orbit coupling controlled by the side gates.

The SO interaction, beyond the undoubtedly important role as it plays in development of spintronics based on semiconductor materials, is also crucial in the physics of topological states of matter [36, 37]. In this context, my latest studies focused on the physics of Majorana states in semiconductor/ superconductor hybrid nanostructures. Although originally the concept of Majorana fermions comes from the physics of elementary particles [38], it is currently one of the most interesting issues in nanotechnology. The increased interest in Majoranas started from the pioneer experiment carried out in the group of prof. Leo Kouwenhoven from University of Delft, who observed quantum states with properties similar to those predicted by Ettore Majorana [39]. These observations were then confirmed independently by other experimentalists [40, 41, 42, 43, 44, 45]. The experiment proposed by Kouwenhoven’s group is the physical implementation of the Kitaev chain [46] and bases on the theoretical studies presented in Refs. [47, 48, 49] in which the authors showed that the \( p \)-wave pairing of spinless fermions occurring in the Kitaev model can be obtained in quasi-one-dimensional systems by the combination of spin Zeeman effect, spin-orbit interaction and \( s \)-type pairing. In the experiment [39], the InSb semiconductor nanowire characterized by the large spin-orbit interaction was covered with NbTiN superconductor inducing the energy gap in the semiconductor material due to the proximity effect. The external magnetic field was applied along the nanowire axis. Above a certain magnetic field, for which the Zeeman energy \( \Delta_Z > \sqrt{\Delta^2 + \mu^2} \), where \( \Delta \) is the superconducting gap and \( \mu \) is the chemical potential, the system undergoes a topological phase transition and the zero-energy Majorana states appear at the ends of nanowire. In the experiment, the presence of Majorana states were observed in the magnetoconductance measurements in the form of the zero bias peak. The topological gap which separates the Majorana states from the higher excited states is determined by the strength of the spin-orbit coupling. This parameter and the ability to control it by external gate potentials is extremely important in the context of future applications of the Majorana states in topological quantum gates. My research in this context concerns determination of the spin-orbit coupling constants in semiconductor nanowires for different gate configurations and geometric parameters [H10].

The increasing interest in the Majorana states is mainly related to their non-abelian statistics and possibilities offered by this property in design of topological quantum gates [50]. However, even the simplest braiding of Majoranas requires at least three-terminal junction. Only recently, the extensive progress in synthesis of semiconductor nanowires with a thin aluminum shell have directed experimental studies towards multiterminal devices [51, 52, 53] which can be used as prototypes for topological quantum gates. Note that in multi-terminal systems, the direction of magnetic field perpendicular to the nanostructure is preferable as it allows for the formation of the Majorana states in each of the branches (Majorana states are formed only when the direction of magnetic field is perpendicular to the Rashba effective field). However, for the perpendicular orientation, the orbital effects start to play an important role. The influence of the orbital effects on the Majorana states in semiconductor/ superconductor hybrid nanostructures is the subject of my latest research included in the presented scientific achievement [H8, H9]. In the paper [H8] we investigated the influence of the orbital effects on the decay length of Majorana states in the weak coupling regime. The second paper [H9] is devoted to the orbital effects in semiconductor/ superconductor hybrids, in the strong coupling regime. In this paper we studied the orbital effects on the induced superconducting gap. The superconducting properties of nanowires under the magnetic field, important in the physics of Majorana states, are the subject of the paper [H3], in which we showed that the orbital effects in superconducting nanowires can induce an unconventional Fulde-Ferrell phase [54] with the non-zero momentum of Cooper pairs.
Methods of calculations

Most of the papers included in my scientific achievement concern computer simulations of the spin-dependent electronic transport in semiconductor nanowires and semiconductor/superconductor hybrid systems. For this purpose, in the papers [H1, H2, H4, H5, H6, H7] I used the one- and multi-band scattering matrix method [55] for Schrödinger and Bogoliubov-de Gennes equation. In a few papers I also used the KWANT package for simulations of quantum transport in low-dimensional systems [56]. The conductivity through the considered nanodevices was determined based on the Landauer approximation or its generalization in the form of the Landauer-Büttiker model [57].

In the papers [H6, H10, H11], in which the Rashba SO coupling constants were determined, I used the 8-band $k \cdot p$ model and the EFA approximation (envelope function approximation). In these papers, the single-electron wave functions were calculated by the use of the Schrödinger-Poisson method, which allows for taking into account the electron interaction in the mean field approximation. In the papers [H3, H9], in which the superconducting properties of nanostructures were studied, I used the BCS model adapted to the considered nanostructures. In Ref. [H3], the BCS model was extended by the possibility of the non-zero momentum pairing (Fulde-Ferrell phase).

Purpose of works [H1–H11]

In my studies on the role of spin-orbit interaction and orbital effects in low-dimensional semiconducting and superconducting structures, I can distinguish the following goals:

- to study the role of spin-orbit interaction and orbital effects in the context of new models of spin filters based on semiconductors,
- to study the role of spin-orbit interaction in the context of the original architecture of spin transistor and the new concepts of spin transistors,
- to investigate the influence of the orbital effects on the Majorana states in semiconductor/superconductor hybrid structures,
- to investigate the influence of the orbital effects on superconducting properties of metallic nanowires.

In the further part of this report, I will discuss the most important results of my studies.

4.2 Discussion of the works that make up the series of publications which are the basis of the request for habilitation


The first of the presented papers is a response to the growing interest in the spin transistors based on 1D semiconductor nanowires, in which the spin relaxation in the conduction channel is strongly reduced by the quantum confinement [26, 27].

In the paper [H1], we considered the spin transistor based on the InAs nanowire with the nearby gate that generates the transverse homogeneous electric field of finite range [Fig. 1(a)]. The Rashba spin-orbit coupling, induced by the electric field, leads to the spin precession of electrons flowing under the gate electrode. In the paper, we used the multi-band transfer matrix method to calculate the spin-dependent transmission coefficients $T_{\sigma' \sigma}$ in the transverse subband $m$ where $\sigma = \uparrow, \downarrow$ are spin indexes. The spin components of current were determined by the use
of the Landauer formula

\[ I_{\sigma\sigma'} = \frac{2e}{h} \sum_{m=1}^{\infty} \int_0^\infty dE T_{m\sigma\sigma'}^m(E) [f(E, \mu_s) - f(E, \mu_d)], \quad (3) \]

where \( f(E, \mu) \) is the Fermi-Dirac distribution function, \( \mu_{s(d)} \) is the chemical potential in the source (drain) and \( M \) is the maximum number of the transverse subbands taken into account.

In the considered model we assumed that the nanowire is connected to ferromagnetic contacts with the spin polarization \( P = (n_\uparrow - n_\downarrow)/(n_\uparrow + n_\downarrow) \) and we neglected the resistance mismatch at the ferromagnet/semiconductor interface. Then

\[ I_\uparrow = \frac{1 + P}{2} I_{\uparrow\uparrow} + \frac{1 - P}{2} I_{\downarrow\uparrow}, \quad (4) \]
\[ I_\downarrow = \frac{1 + P}{2} I_{\uparrow\downarrow} + \frac{1 - P}{2} I_{\downarrow\downarrow}. \quad (5) \]

The total current is given by

\[ I = \frac{1 + P}{2} I_\uparrow + \frac{1 - P}{2} I_\downarrow. \quad (6) \]

We studied the spin transistor operation for two cases: (A) the full spin polarization of electrons in the contacts \( P = 1 \) using the one-subband approximation, and (B) the partial spin polarization of electrons in the contacts \( P \neq 1 \) in the many-subband approximation.

![Figure 1](image-url)

Figure 1: (a) Schematic of the nanowire spin transistor. Dispersion relations \( E(k) \) for source-gate, gate, and gate-drain regions of the nanowire are presented in the upper part of the illustration. (b) Spin density \( s_z \) as a function of the gate voltage \( V_g \) and coordinate \( z \). The electrons with spin \( s_z = +\hbar/2 \) are injected from the source at \( z = 0 \) and precesses in the region of gate marked by the two horizontal dashed lines. The spin state of electrons in the drain, at \( z = 250 \, \text{nm} \), oscillates as a function of \( V_g \). (c) The total current \( I/I_0 \) as a function of the gate voltage \( V_g \) with marked ’on’ and ’off’ states. \( I_0 = 2e\mu/h \). [H1].

The case (A) is the idealized model of the spin transistor with the spin polarization of source and drain, \( P = 1 \). We considered the single-band model in which electrons occupy the lowest subband (ground state). For the assumed spin polarization of contacts \( P = 1 \), the spin-up
electron is injected into the nanowire from the ferromagnetic source, at \( z = 0 \). The electronic transport through the nanowire is assumed to be ballistic. The spin of electrons flowing through the nanowire rotates in the region of the electric field due to the Rashba spin-orbit coupling. Fig. 1(b) displays the \( z \)-component of the electron spin as a function of the position and the gate voltage. We see that the length of precession decreases with increasing the gate potential - the spin rotates a greater number of times at the same length which approximately corresponds to the gate length. As a result [Fig. 1(b)] the spin of electrons at the output (\( z = 250 \) nm) oscillates as a function of the gate potential. For the full spin polarization of the drain, \( P = 1 \), electrons with spin opposite to the polarization of drain cannot be absorbed and are backscattered. As a result, the total current flowing through the transistor oscillates as a function of the gate voltage - Fig. 1(c). We see that for the gate voltages, for which the electron spin at the output has been reversed, the total current decreases to zero. As we will show below, this takes place only for the ideal case, where \( P = 1 \), without spin relaxation in the conduction channel. The states 'on' and 'off' are defined in the maximum and minimum of conductivity. The slight decrease of the current in Fig. 1(c) results from the electric field in the gate region, which leads to the partial reflection of electrons flowing through the nanowire.

![image]

Figure 2: (a) Normalized current \( I_n = I/I_{\text{max}} \) as a function of the source-drain voltage \( V_{sd} \) for different gate voltages \( V_g \). Comparison with the experimental data taken from Ref. [25]. The calculations were carried out for the polarization of electrons in the contacts \( P = 0.4 \). \( I_{\text{max}} = I(V_{ds} = 1.0V, V_g = 0) \). (b) Current \( I \) as a function of the gate voltage \( V_g \) for \( V_{sd} = 0.6 \) V. Upper panel – results of calculations, lower panel – experimental data. The calculations were carried out for \( P = 0.4 \) [solid (red) curve] and \( P = 1 \) [dashed (blue) curve]. \( I_0 = 2e\mu/h \). [H1].

Calculations performed for the more realistic model (B) with the finite spin polarization of contacts and the electron transport in many subbands were motivated by the experiment performed by the Japanese group of prof. J. Yoh [25]. This experiment presents the electronic transport measurements in the spin transistor architecture based on the InAs nanowire with a wide gate and contacts made of Fe. In our calculations, we assumed geometrical parameters in accordance with the experiment and the spin polarization of contacts \( P = 0.4 \) which corresponds to the spin polarization of electrons at the Fermi level for Fe. In the model, the only fitting parameter was the chemical potential \( \mu \). As shown in Fig. 2, our results agree with the experimental data. The most important result is shown in Fig. 2(b), which displays the oscillations of current as a function of the gate potential. We see that the inclusion of the finite contact polarization (red line) leads to the strong reduction of the oscillation amplitude. In addition, the use of geometry with the wide gate leads to the increase of the current as a function of \( V_g \), clearly apparent on the background of the oscillations.
The proposed spin filter is based on the Y-shaped nanostructure with QPC localized in one of the branches [Fig. 3(c)]. In the considered model, the unpolarized charge current injected from the contact 1 is separated in the QPC region into two spin-polarized currents flowing out through the contacts 2 and 3. The potential energy in the nanostructure can be separated into the sum of the confinement potential and the potential from QPC which is taken in the following form

$$U_{QPC}(x, y) = \frac{1}{2} m_e \omega^2 y^2 \exp \left[ \frac{-(x - x_0)^2}{2d^2} \right],$$

where $d$ determines the size of QPC, $x_0$ defines the position of its center, and $\hbar \omega$ is the energy of the transverse parabolic confinement in the QPC region. The nanostructure is subjected to the magnetic field $B = (0, 0, -B)$ perpendicular to the plane of the nanostructure. In the proposed model, the spin separation effect results from the combined Zeeman spin-splitting and orbital effects which guarantee that the beam of spin polarized electrons, reflected from QPC, is directed to the opposite branch of the nanostructure.

In the paper, based on the Landauer-Büttiker formula, we determined the spin-dependent conductance between the contacts as a function of the QPC confinement energy and the magnetic field (Fig. 4). Figures 4 (a) and (b) show that the rapid decrease of the conductance between the contacts 1 and 2 for the spin-up electrons occurs for the higher QPC confinement energy than for the spin-down electrons. Simultaneously, for both the spins, the decrease of the electron transmission into the channel 2 is accompanied by the increase of the transmission into the channel 3. This means that in the confinement energy regime, for which the conductance of the spin-up electrons through the channel 2 is still large, the spin-down electrons are reflected from QPC and flow out through the channel 3. As a result, the spin-unpolarized current injected from...
the contact 1 splits into the two spin-polarized beams. The spin splitting is clearly demonstrated in Fig. 4(c,d) which show the spin conductances defined as $P_{nm} = G_{nm}^\uparrow - G_{nm}^\downarrow$ as a function of the QPC confinement energy, $\hbar \omega$.

The spin separation effect in the proposed nanostructure can be explained as follows. In the external magnetic field, the spin degeneracy of the transverse electron states in the nanostructure is lifted due to the spin Zeeman effect. We assume that the non-polarized electron beam is injected into the system from the contact 1 with the chemical potential being adjusted so that only the two lowest-energy transverse modes are active in the electron transport (the first corresponding to spin-up electrons and the second corresponding to spin-down electrons) [Fig. 3(b)]. Note that the energies of electronic states in the QPC regions can be controlled by the QPC confinement energy $\hbar \omega$, experimentally implemented by changing the potentials applied to the gates. In particular, $\hbar \omega$ can be set so that the chemical potential in the QPC region is located between the spin-split electron states as shown in Fig. 3(a). For such a value, the transmission coefficient of spin-up electrons into the contact 2 is still large, while the spin-down electrons are reflected from QPC due to the lack of available states in this region. The spatial separation of the incident and reflected beam is supported by the orbital effect. The electrons flowing in the $\pm x$ direction are always shifted by the Lorentz force to the right boundary of the conduction channel (the right and left sides are defined with respect to the direction of the current flow). In other words, the Lorentz force pushes the electrons to the right edge of the conduction channel and prevents the spin-down electrons reflected from QPC to flow back into the channel connected with the contact 1.

The spin separation effect is shown in Fig. 4(e) which presents the charge and spin density distributions for three different values of the QPC confinement energy: (a) $\hbar \omega = 5$ meV when both the spin-up and spin-down electrons are transmitted through QPC, (b) $\hbar \omega = 14$ meV for which we observe the spin separation effect and (c) $\hbar \omega = 20$ meV for which the QPC region ceases to be transparent for both the spin components. The slight spin polarization of current injected from the contact 1, shown in Fig. 4(e), results from different density of states at the Fermi level for spin-up and spin-down electrons induced by the spin Zeeman effect.

In the paper [H2] we also analyzed the impact of the spin-orbit interaction on the proposed spin separator. Calculations with the spin-orbit coupling taken into account, at the level of experimentally measured in heterostructures In$_{0.5}$Ga$_{0.5}$As, showed that the SO interaction does not change the spin separation effect in a significant manner - in Fig. 5(a) we present $P_{nm}(\hbar \omega)$.
Figure 5: Spin conductance $P_{nm} = G_{nm}^+ - G_{nm}^-$ as a function of $\hbar \omega$. (a) Results with the spin-orbit interaction taken into account, for two different Rashba coupling constants, $\alpha$. (b) Results with the inclusion of the scattering within the Ando model for different mean free path, $\ell$. [H2].

From the applications point of view, it is much more advantageous that the proposed spin separator model is robust against the scattering and therefore it also works in the non-ballistic regime. This property results from the characteristics of electron transport in the external magnetic field which for sufficiently large $B$ is carried by the edge states. Electron beams flowing in opposite directions are located on opposite sides of the conduction channel - see Fig. 4(e). As a result, backscattering in the channel is strongly suppressed because the change of the wave vector to the opposite requires a change of the quantum state to that localized on the opposite side of the channel. The probability of such a process is very low due to the small overlap between the states located on opposite sides of the nanowire. This mechanism is well known and is the origin of the "zero" resistance in the quantum Hall effect [55].

In order to obtain quantitative estimation of the scattering effects, we included the spin-independent scattering in the framework of the Ando model [58] by assuming that the on-site potential energy is uniformly distributed within the range $[-W/2, W/2]$. The relation between energy $W$ and the mean free path $\ell$ was taken from Ref. [58] and is given by

$$W_{EF} = \left( \frac{6 \lambda_F^2}{\pi^3 a^2 \ell} \right)^{1/2},$$

where $E_F$ is the Fermi energy, $\lambda_F$ is the Fermi wave length and $a$ is the lattice constant.

The calculations were carried out for the values of the mean free path corresponding to the experimentally measured for 2DEG in InGaAs ($\ell \approx 1 \mu m$) [59]. Figure 5(b) shows the spin conductance $P_{12}$ and $P_{13}$ with the spin-independent scattering taken into account, as a function of $\hbar \omega$ for different values of the mean free path $\ell$. The results presented in Fig. 5(b) were obtained by averaging over $10^4$ computational runs with different random values of the on-site energies for each fixed energy $\hbar \omega$. Figure 5(b) shows that the spin-independent scattering only slightly affects the spin-splitting effect - it decreases only slightly with decrease of the mean free path in the realistic range corresponding to the experimental measurements [59].


The paper [H3] concerns the superconducting properties of cylindrical nanowires and is part of the study devoted to the Majorana states in semiconductor/superconductor hybrid nanostructures. The formation of Majorana states in semiconductor nanowires with a strong SO interaction requires the electron pairing which in the experimental setup is induced by the proximity effect. For this purpose, in recent experiments [60] a thin layer of Al is epitaxially
deposited on the two facets of semiconductor nanowire. Therefore, the superconducting properties of metallic nanostructures are crucial in the experimental implementation of the Majorana states. In this respect, it is also relevant to study the superconducting properties under the external magnetic field as the presence of the magnetic field is one of the necessary conditions for the Majorana states formation. In the paper [H3] we studied the superconducting properties of thin metallic Al nanowires in the external magnetic field directed along the nanowire axis.

Figure 6: (a) Schematic representation of Cooper paired states in the nanowire. Top: BCS pairing, bottom: FF pairing. The horizontal arrow connects the pair partners, for which the sets of quantum numbers are specified in the brackets. (b) Superconducting critical temperature $T_c$ as a function of the nanowire radius $R$ for $H_{||} = 0$. [H3].

The Fulde-Ferrell (FF) phase [54] is the unconventional phase of superconductivity in which the total momentum of the Cooper pair is non-zero, in contrast to the BCS phase. Previous studies on the FF phase stability conditions clearly show that the FF phase can be induced by the magnetic field in systems in which the orbital effects are negligibly small [61], such as heavy fermion compounds [62, 63, 64] or organic superconductors [65, 66]. In our paper [H3] we showed that the orbital effects in superconducting nanowires, not only do not destroy the FF phase, but can be direct source of pairing with the non-zero total momentum of Cooper pairs. The proposed model sheds new light on the impact of the orbital effects on the FF phase.

In our paper [H3] we considered the superconducting metallic nanowire with cylindrical symmetry in the presence of the external magnetic field directed along the nanowire axis. In the considered system, the single electron states are determined by four quantum numbers $(m, j, k, \sigma)$, where $m$ is the orbital quantum number, $j$ is the radial quantum number, $k$ it the wave vector along the nanowire axis and $\sigma$ is the electron spin. The single-electron energy can be expressed as

$$\xi_{k,m,j} = \frac{\hbar^2 \gamma_{m,j}^2}{2m_e R^2} + \frac{\hbar^2 k^2}{2m_e} + m\mu_B H_{||},$$ (9)

where $\gamma_{m,j}$ is the $j$-th zero of the $m$-th order Bessel function, $R$ is the nanowire radius and $H_{||}$ is the magnetic field intensity. Note that the Zeeman effect was deliberately not taken into account to emphasize the role of the orbital effects in the FF phase formation (the influence of the spin Zeeman effect will be discussed in the further part of this report).

In the BCS model, Cooper pairs are formed by pairing of electrons with opposite spins from opposite sides of the Fermi surface $(k, \uparrow) \leftrightarrow (-k, \downarrow)$ so that the total momentum of Cooper pairs is zero. In the model under consideration the BCS pairing occurs between the electrons in states $(m, j, k, \uparrow) \leftrightarrow (-m, j, -k, \downarrow)$. Note, however, [Eq. (9)] that the external magnetic field lifts degeneracy with respect to the orbital quantum number, $m$. In the magnetic field, the electron states with opposite $m$ have the energy shifted by $2m\mu H_{||}$ from each other. The Fermi vector mismatch between the states $\pm m$ can be compensated by the non-zero momentum of the Cooper pair, $q$. This leads to the formation of the FF phase, in the considered system induced solely by the orbital effects. The BCS and FF pairing scheme is schematically depicted in Fig. 6(a).
Figure 7: (a) Superconducting gap $\Delta$ as a function of the magnetic field $H_{||}$. The stability regions of the FF phase are marked by the gray areas. (b) Total Cooper pair momentum which minimizes the energy vs. $H_{||}$. (c) Cooper pair momentum dependence of the free energy in the paired and normal states for selected value of the field. The energy minimum visible in (c) corresponds to the stability of the FF phase. Results for $R = 0.97$ nm labeled by II in Fig. 6(b). (d) Phase diagrams $\Delta(H_{||})$ for different nanowire radii $R$ labeled by (I)-(IV) in Fig. 6(b). Results with the Zeeman effect taken into account. [H3].

In the paper [H3], the energy gap in the external magnetic field was determined based on the BCS model adopted for nanowires with defined symmetry, taking into account the non-zero total momentum pairing with $q \neq 0$. Fig. 6(b) presents the critical temperature $T_c$ as a function of the nanowire radius $R$ for $H_{||} = 0$. Peaks of the critical temperature results from the quantum size effect. In the BCS model, the value of $T_c$ (and the superconducting gap) depends on the density of states near the Fermi level. In quasi-1D nanoscopic systems, the density of states increases rapidly in the van Hove singularities [67] that occur each time when the bottom of the subband passes through Fermi level. For the radius $R$ which supports the van Hove singularity occurrence, we observe a peak in the critical temperature.

Figure 7(a) shows the superconducting energy gap $\Delta$ as a function of the magnetic field $H_{||}$, calculated for the nanowire radius $R$ labeled by (II) in Fig. 6(b). We can see that the superconductor-to-normal metal transition driven by the magnetic field occurs as a cascade of jumps in the order parameter. Each jump corresponds to the Cooper pair breaking in a subsequent subband whose coupling with the magnetic field depends on the orbital quantum number $m$. Figure 7(a) shows that the Fermi wave vector mismatch for the subbands $\pm m$ can be compensated by the non-zero momentum of Cooper pairs leading to the alternating BCS and FF phases. In Fig. 7(b) we present the $q$ vector as a function of the magnetic field. The value of $q$ was determined by minimizing the free energy of the system. The minimum of the free energy for the non-zero $q$ (FF phase) is displayed in Fig. 7(c).

Our further calculations showed that the FF phase occurs only for nanowire radii $R$ which satisfy certain conditions - see Fig. 7(d). Namely, the FF phase appears only if the dominant quasi-particle branch has the orbital quantum number lower than the other superconducting branches excluding those with $m = 0$. In addition, we showed that the similar phase diagrams with alternating BCS and FF phases are obtained for a realistic model with the spin Zeeman effect - see Fig. 7(d).

The proposed mechanism of the FF phase formation induced by the orbital effects appears
only if few states with different quantum numbers participate in the superconducting phase. At the same time, the thicker the nanowire is, the more states take part in the pairing. As a consequence, our analysis is restricted to relatively thin nanowires. Nevertheless, by lowering the electron concentration we also decrease the number of states taking part in the pairing. This fact allows us to suggest that for superconductors with low concentration of carriers, e.g., for SrTiO, the presented effect may be observed also for much thicker nanowires.


As mentioned in the section 4.1, the current research on the spin transistor is conducted in two directions. One of them relies on searching for new spin transistor concepts, different from that proposed in 1990 by Datta and Das [5]. An alternative spin-transistor design with the spin signal observed over the distance 50 µm has been recently described by Žutić and Lee [68] and experimentally demonstrated by Betthausen et al. in Ref. [34]. In this approach, the spin control is realized by combining the homogeneous and helical magnetic field. The latter is generated by ferromagnetic stripes located above the conduction channel (Cd,Mn)Te. The transistor action is driven by the diabatic Landau-Zener transitions [35] induced by the appropriate tuning of the homogeneous magnetic field. For these suitably chosen conditions the backscattering of spin polarized electrons appears, which gives raise to the large increase of the resistance, i.e., the transistor goes over into the ‘off’ state.

![Figure 8](image-url)

Figure 8: (a) Potential energy profiles for different values of $\gamma = B_{ext}/B_h$. $E$ is the minimum energy of electron in the subband $\varphi^\pm$ ( + and – denotes the state with spin parallel and antiparallel to the magnetic field). For $B_{ext} = B_h$ ($\gamma = 1$) the Landau-Zener transition probability $P = 1$. For these suitably chosen conditions the backscattering of spin polarized electrons appears, which gives raise to the large increase of the resistance (‘off’ state). (b) Schematic illustration of the resonant Landau-Zener transition leading to the backscattering and additional conductance dips for $B_{ext} \neq B_h$. [H4].

Our paper [H4] concerns the spin-dependent electronic transport in the new spin transistor architecture proposed in Ref. [34]. For this purpose, we considered a two-dimensional nanowire with (Cd,Mn)Te in the presence of the magnetic field being a combination of the homogeneous magnetic field $B_{ext} = (0,0,-B_{ext})$ and the helical field $B_h$ (in the experiment generated by ferromagnetic stripes)

$$B_h(r) = B_h \left( \sin \frac{2\pi x}{a}, 0, \cos \frac{2\pi x}{a} \right),$$

where $B_h$ is the amplitude of the helical field and $a$ is the period of the magnetic field modulation.

In the considered model, the energy of the lowest state with spin parallel $E^+$ and anti-parallel
where \( \gamma = B_{\text{ext}} / B_h \). Parameter \( \gamma \) can be tuned by changing the homogeneous magnetic field \( B_{\text{ext}} \) and determines the spatial modulation of the spin-split subbands as depicted in Fig. 8(a).

In particular, for \( \gamma = 1 \) the probability of the Landau-Zener transition in the central part of the nanodevice, \( P = 1 \). Note, however, that electrons initially injected into the system in the state \( \varphi^- \), after the Landau-Zener transition to the state \( \varphi^+ \), are backscattered since the energy of the state \( \varphi^+ \) in the right contact is higher than the Fermi energy - see Fig. 8(a). This leads to the conductance dip at \( B_{\text{ext}} = B_h \). An important element of the proposed model is the fact that the spin state of electrons flowing through the channel is protected against a possible decay by keeping the transport in the adiabatic regime. In the considered nanostructure, the adiabaticity can be defined by the parameter \( Q = \omega_L / \omega_{\text{mod}} \), where \( \omega_L = g_{\text{eff}} \mu_B B_h / \hbar \) is the frequency of the spin Larmor precession and \( \omega_{\text{mod}} = 2 \pi v_F / a \) is the frequency of the magnetic field modulation measured in the electron rest frame, where \( v_F \) is the Fermi velocity. In the adiabatic regime \( Q \gg 1 \).

In the paper [H4] we analyzed the spin transistor operation in the range of parameters in which the condition of adiabaticity is not fully met. For this purpose, we performed the simulations of spin-dependent electronic transport for the various helical field modulation period which determines the parameter \( Q \). Figure 9 presents the conductance \( G \) as a function of the Fermi energy \( \mu_F \) and the homogeneous magnetic field \( B_{\text{ext}} \), calculated for three different values of \( a \). The corresponding value of \( Q \) is placed in the panels. In our calculations, the energy range \( \mu_F \) was chosen so that the electron is injected into the nanodevice in the lower band \( \varphi^- \).

The apparent dip in conductance for \( B_{\text{ext}} = B_h \) (we assumed \( B_h = 50 \) mT) results from the Landau-Zener transition and the corresponding backscattering. Note, however, that for the low values of \( Q \) [Fig. 9(a)], we observe the additional conductance dips for \( B_{\text{ext}} \neq B_h \). The analysis of the conductance maps in Fig. 9 allows us to extract two characteristic features of this effect: (i) the period of the occurrence of the additional dips (measured as a function of the Fermi energy) decreases with the increasing period \( a \) of the magnetic field modulation, (ii) the additional conductance dips disappear in the adiabatic regime (we see that for \( a = 8 \) \( \mu \)m \( (Q \approx 25) \) the additional conductance dips are strongly suppressed). The explanation of this phenomenon is described in detail in the paper [H4] based on the analysis of the electron density distributions and the spatially dependent inter-subband transitions coefficients. In brief, the effect is related to the resonant Landau-Zener transitions and can be described as follows.

For the short period \( a \), the magnetic field, acting on the electron in its reference frame, changes so fast that the electron spin cannot adapt to changes of the magnetic field. This means that the electron, initially injected into the lowest subband \( \varphi^- \), does not remain in this subband.
when flowing through the nanostructure. The quantum state of the electron can be described by the linear combination of the eigenstates with the spin orientation parallel and antiparallel to the magnetic field. Since the wave functions of this state \( \psi(x) = c_+(x)\varphi^+(x) + c_-(x)\varphi^-(x) \), the probability of the inter-subband transition is non-zero. If the energy of the injected electron is higher than \( E_{\text{min}} \), defined as the minima of the band \( \varphi^+ \) [see Fig. 8(a)], the electron flowing through the nanostructure experiences the effective quantum well created in the spatially varying subband \( \varphi^+ \). In this quantum well, the quasi-bound states are created. If the Fermi energy becomes equal to the energy of the quasi-bound state formed in the subband \( \varphi^+ \), the probability of the inter-subband transition approaches \( P = 1 \) - this process is called the resonant Landau-Zener transition. As a result the electrons injected into the nanowire are transmitted to the subband \( \varphi^+ \). Since the energy of the subband \( \varphi^+ \) in the right contact is higher than the Fermi energy, the electrons are backscattered which gives raise to the additional conductance dips. This mechanism is schematically illustrated in Fig. 8(b).

In order to confirm the proposed interpretation, we calculated the energy of quasi-bound states in the effective quantum well that is formed in the band \( \varphi^+ \). The determined energies, marked by white dashed lines in Fig. 9(b), exactly correspond to the occurrence of the additional dips in the conductance. This finally confirmed the proposed explanation based on the resonant Landau-Zener transitions.


Although the new architecture of spin transistor [34] seems to be robust against the spin relaxation processes, it has a one serious disadvantage, namely the transistor action in the proposed nanodevice is induced by the external magnetic field. The precise application of the magnetic field in a small area is not easy from the experimental point of view and is a serious limitation in the context of applications of this type of transistors in the integrated electronic circuits. For this reason, in the paper [H5] I proposed a model of the spin transistor based on a helical magnetic field, in which the transistor action is driven by the electric field from the side gates. The idea behind this solution is to replace the external magnetic field with the Rashba field. The schematic model of the proposed transistor is shown in Fig. 10.

![Figure 10](image)

**Figure 10:** (a) Schematic of the proposed spin transistor. InSb nanowire grown in the [111] direction is subjected to the helical magnetic field generated by ferromagnetic stripes located near the conduction channel. Rashba SOI is controlled by the side gates \( g_1 \) and \( g_2 \). (b) Profile of the helical magnetic field and spin distribution. [H5].

In the considered model, the InSb nanowire, similarly as in the experiment, is located on ferromagnetic stripes which generate the helical field

\[
B_h(x) = B_h \left( \sin \frac{2\pi(x - x_0)}{a}, 0, \cos \frac{2\pi(x - x_0)}{a} \right),
\]

(12)
where $B_h$ is the helical field amplitude, $a$ is the period of the magnetic field modulation determined by the distance between ferromagnetic stripes and $x_0$ is the localization of the ferromagnetic stripe. For simplicity, we study the nanowire with length $L$ equal to one period of the helical magnetic field modulation, i.e., $L = a$ and $x_0 = L/2$.

The spin dynamics in the nanostructure is determined by the effective magnetic field being the sum of the helical field and the Rashba field. In the adiabatic regime, the spatially dependent single-electron energy, parallel $E^+$ and anti-parallel $E^-$ to the effective field, is given by

$$E^\pm(x) = \pm \alpha k_x \sqrt{1 + \gamma^2 + 2\gamma \cos \left( \frac{2\pi(x-x_0)}{a} \right)},$$  \hspace{1cm} (13)$$

where $\gamma = \frac{\beta g_{eff} \mu_B B_h}{\alpha k_x}$. The parameter $\gamma$ depends on the Rashba coupling constant $\alpha$ which is controlled by the electric field generated the potential applied to the side gates.

Note that the physical picture in which the electron moves in the spatially-dependent magnetic field can be replaced by the picture in which the electron is subjected to the time dependent perturbation that comes from the effective magnetic field. For the considered two-level system subjected to the time-dependent perturbation, the probability of the Landau-Zener transition is given by

$$P = \exp \left( -\frac{2\pi \varepsilon_{12}}{\hbar^2} \frac{12}{\beta} \right),$$  \hspace{1cm} (14)$$

where $\varepsilon_{12} = \alpha k_x |\gamma - 1|$ corresponds to half the closest distance between the energies $E^\pm$ at the crossing point. Parameter $\beta = \frac{1}{\hbar^2} \frac{d}{dt} (E^+ - E^-)$ determines how fast these eigenenergies approach each other in the electron rest frame. Using the relation $\partial E/\partial t = (\partial E/\partial x)(\partial x/\partial t) = (\partial E/\partial x)V_F$, where $V_F$ is the Fermi velocity, $\beta$ takes on the form

$$\beta = \frac{2\pi g_{eff} \mu_B B_h V_F}{a \hbar} \frac{2 \sin \left( \frac{2\pi(x-x_0)}{a} \right)}{\sqrt{1 + \gamma^2 + 2\gamma \cos \left( \frac{2\pi(x-x_0)}{a} \right)}}. \hspace{1cm} (15)$$

We can see that the Landau-Zener transition probability depends on the energy distance between $E^\pm$ states, which in turn is determined by the SO coupling. Fig. 11(a) presents the probability of the Landau-Zener transition $P$ as a function of the Rashba spin-orbit coupling constant $\alpha$ for two different values of the Fermi vector $k_F$. We see that the maximum of $P$ moves towards the higher $\alpha$ as $k_F$ increases.

We can switch the system into the high resistance state (‘off’ state) by a suitably electric field.
the gate potentials). It should be noted that the current experimental papers on nanowires with side gates [69], unambiguously indicate the possibility of experimental implementation of the proposed spin transistor model.


The paper [H6] refers to the spin transistor based on the coupled quantum nanowires, where besides the intra-subband SO interaction, the inter-subband SO coupling appears in the system. The inter-subband SO coupling, although often overlooked in theoretical studies, is the source of many interesting phenomena including the anomalous Zitterbewegung effect [70] or the internal spin Hall effect in symmetric quantum wells [71]. In my paper [H6] we studied the influence of the inter-band SO interaction on the operation of the spin transistor based on the coupled quantum wells.

In the proposed spin transistor model, the conduction channel in the form of coupled 2D nanowires (quantum wells) is located between two ferromagnetic contacts acting as a spin injector and detector. For simplicity, we assumed that the spin injection (detection) efficiency in the ferromagnetic contacts reaches 100 %. Fig. 12(a) presents the schematic of the considered spin transistor model together with the cross-section of heterostructure in the grown direction - panel (b). We considered $\text{Al}_{0.48}\text{In}_{0.52}\text{As}/\text{Ga}_{0.47}\text{In}_{0.53}\text{As}$ double quantum well (width 50 nm) with the central barrier $\text{Al}_{0.3}\text{In}_{0.7}\text{As}$ with width $w_b$ which determines the coupling between the conduction electron states in the quantum wells. The nanostructure contains two $n-$doped layers with donor concentrations $N_d = 4 \times 10^{18}$ cm$^{-3}$ located on either side of the quantum well. The Rashba SO interaction can be tuned by the external gates with the length $L_g$ located below and above the quantum well. By applying the suitably chosen gate potentials the spin transistor can be electrically switched between the ‘on’ and ‘off’ states as shown in Fig. 12(e).

Simulations of the spin-dependent electronic transport through the considered nanostructure were performed by the use of the two-band scattering matrix method which allows to determine the transmission coefficients $T_{\sigma \sigma'}^{nm}$, where $\sigma, \sigma' = \uparrow, \downarrow$ are spin indexes and $n, m = 1, 2$ is the subband number. If we assume that the spin polarization in contacts $P = 1$, the conductance is
given by the Landauer formula

\[ G = \sum_{n,m=1}^{2} G_{nm}^{\uparrow\uparrow} = \frac{e^2}{h} \int T_{nm}^{\uparrow\uparrow}(E) \left( \frac{\partial f_{FD}(E, E_F)}{\partial E} \right) dE, \]  

where \( f_{FD} \) is the Fermi-Dirac distribution and \( E_F \) is the Fermi energy.

\[ \frac{\Delta k}{k_F} = \frac{\Delta k_{\downarrow}}{k_{\downarrow}} \]

Let’s start from the two-band model with the energy distance between the bands \( \Delta \varepsilon = \varepsilon_1 - \varepsilon_2 \), in which the SO coupling constants are treated as the simulation parameters. Moreover, we assume \( \alpha_{11} = \alpha_{22} = \alpha \). In Fig. 13(a) we present the conductance \( G \) as a function of the intra- (\( \alpha \)) and inter-subband (\( \alpha_{12} \)) SO coupling constant. As expected, for \( \alpha_{12} = 0 \) [Fig. 13(b)] the conductance oscillates as a function of \( \alpha \) according to the formula

\[ G = 2G_0 \cos^2 \left( \frac{\Delta k L_g}{2} \right) = 2G_0 \cos^2 \left( \frac{m^* \alpha L_g}{\hbar^2} \right), \]  

where \( \Delta k = k_{\uparrow} - k_{\downarrow}, m^* \) is the effective mass and \( L_g \) is the length of the gate. The conductance oscillations result from the electron spin precession, independently in each of the bands, around the effective Rashba field as shown in Fig. 13(b) (see the inset). The conductance maximum corresponds to the integer rotation number of the electron spin while the minimum is related to the half-integer rotation number - then, the electron flows out of the conduction channel within the spin-down state which can not be absorbed by the spin-polarized drain (\( P = 1 \)) and it is backscattered.

Regular conductance oscillations are considerably modified when we introduce the inter-subband SO coupling, \( \alpha_{12} \neq 0 \). Then, the probability of the inter-subband transitions with spin flip becomes non-zero. Fig. 13(a) and (b) show that within a certain range of \( \alpha_{12} \), the inter-subband SO coupling modifies the oscillation period only slightly, leading rather to a small decrease in their amplitude - results for \( \alpha_{12} = -10 \text{ meVnm} \). Nevertheless, there is a certain value of the parameter \( \alpha_{12} \) for which the conductance oscillations undergo a phase shift - results for \( \alpha_{12} = -18 \text{ meVnm} \). Note that, even with the intra-subband SO constant \( \alpha = 0 \) the conductance oscillates as a function of the inter-subband SO coupling \( \alpha_{12} \). In this case, the spin precession length \( \lambda_{SO} \) can be determined analytically

\[ \lambda_{SO} = \frac{2\pi}{\Delta k} = \frac{2\pi}{k_2 - k_1}, \]
where

\[ k_1 = \sqrt{\frac{2m^*E_F}{\hbar}} \sqrt{1 - \frac{\varepsilon_+ - \varepsilon_-}{E_F} \left(1 + \left(1 + \frac{2\hbar^2(E_F - \varepsilon_-)}{\alpha_{12}^2 m^*} + \frac{\hbar^4 \varepsilon_-^2}{\alpha_{12}^4 m^*^2}\right)\right)}, \]

\[ k_2 = \sqrt{\frac{2m^*E_F}{\hbar}} \sqrt{1 - \frac{\varepsilon_+ - \varepsilon_-}{E_F} \left(1 - \left(1 + \frac{2\hbar^2(E_F - \varepsilon_-)}{\alpha_{12}^2 m^*} + \frac{\hbar^4 \varepsilon_-^2}{\alpha_{12}^4 m^*^2}\right)\right)}, \]

with \( \varepsilon_+ = (\varepsilon_1 + \varepsilon_2)/2 \).

Our study of the inter-subband SO coupling on the conductance was based on the analysis of electron and spin density distributions in both the bands and is presented in detail in Ref. [H6]. In the paper [H6] we additionally investigated the impact of asymmetry associated with different Rashba couplings in both the bands, \( \alpha_{11} \neq \alpha_{22} \).

In the second part of the paper [H6] we considered the realistic structure based on Al\(_{0.48}\)In\(_{0.52}\)As/Ga\(_{0.47}\)In\(_{0.53}\)As [Fig. 12(a, b)]. The Rashba coupling constants were determined based on the 8 band \( \mathbf{k} \cdot \mathbf{p} \) model taking into account the electron interaction in the mean field approximation (Schrödinger-Poisson method). The Dresselhaus constant was estimated based on the expression

\[ \beta_n = \beta^{3D} \langle \varphi_n | \hat{k}_z^2 | \varphi_n \rangle, \]

(21)

where \( \beta^{3D} = 0.0237 \text{ meVnm}^3 \) [72].

Figure 14: Intra-subband (a) \( \alpha_{11} \), (b) \( \alpha_{22} \) and (c) inter-subband \( \alpha_{12} \) Rashba SO coupling constant as a function of the gate voltage \( V_g \) for different electron concentrations \( n_e \). (d) Dresselhaus SO constant \( \beta(V_g) \) for \( n_e = 10 \times 10^{10} \text{ cm}^{-2} \). [H6].

Fig. 14 presents the Rashba SO constants as a function of the gate potential \( V_g \) for different electron concentrations \( n_e \). We see that in the gate voltage range under consideration, the Dresselhaus SO constant \( \beta \) is two orders of magnitude smaller than the Rashba constants. Therefore, the Dresselhaus SO interaction will be neglected in the further part of the consideration. Fig. 14(c) shows that the inter-subband SO constant is an even function of the gate voltage and exhibits the resonant behavior around \( V_g = 0 \). At the same time, the intra-subband constants \( \alpha_{11}, \alpha_{22} \) change rapidly at \( V_g = 0 \), which is especially pronounced for high electron concentrations. The rapid change of the Rashba constant around \( V_g = 0 \) makes the considered system favorable from the applications point of view where the transistor action should be induced in the voltage range as narrow as possible. For the determined electronic structure and the coefficients \( \alpha_{nm} \), we calculated the conductance \( G \) as a function of the gate voltage \( V_g \), presented in Fig. 15(a). As we see, the conductance exhibits maximum at \( V_g = 0 \) (low resistance state), and a slight change of \( V_g \) switches the system into the high resistance state (’off’ state). The ratio \( G_{on}/G_{off} \) depends on the electron concentration and is the highest for high values of \( n_e \).
Note that in the proposed spin-transistor model, the transistor action occurs in a narrow voltage range around $V_g = 0$, in which the inter-subband Rashba coupling constant reaches its maximum [Fig. 14(c)]. The influence of the inter-subband SO coupling can be investigated by analyzing the conductance maximum at $V_g = 0$ for which the intra-subband SO constants $\alpha_{11} = \alpha_{22} = 0$. As shown in the first part of the paper [H6], the conductance $G = 2G_0$ at $V_g = 0$. However, Fig. 15(a) shows that $G = 2G_0$ at $V_g = 0$ is obtained only for low electron concentrations, for which the inter-subband SO interaction is negligible. Thus, a decrease of $G(0)$ with increasing $n_e$, shown in Fig. 15(b), corresponds to the increase of the inter-subband SO interaction. For $V_g \approx 0$, it induces the inter-subband transitions with spin flip, which in turn leads to decrease of the conductance. The probability of such a transition depends not only on $\alpha_{12}$, but also on the energy distance between the bands $\Delta \varepsilon$ shown in Fig. 15(c).

In conclusion, the presence of the inter-subband SO interaction leads to decrease in the conductance at $V_g = 0$ and the reduction of the ratio $G_{on}/G_{off}$, important from the applications point of view. The undoubted advantage of the proposed model is the fact that for high concentrations $n_e$, the transistor action can be induced by a voltage change in a narrow range around $V_g = 0$.


The experimental realization of an effective spin transistor based on the coupled quantum wells [H6] requires to design compatible spin filters based on semiconductor materials that allow for an efficient spin injection and detection (let us remind that it is not possible by the use of ferromagnetic contacts due to the resistance mismatch between ferromagnet and semiconductor). For this reason, in the paper [H7] we proposed the spin filter based on the coupled quantum wires with the quantum point contact (QPC), in which the spin filtering effect is induced by the inter-subband SO interaction.

We considered the bilayer nanowire consisting of two coupled conducting channels of width $W$ with QPC located in the middle of the nanowire. The lateral electric field $\mathbf{F} = (0, F_y, 0)$ was applied to the system, which as shown by the recent experiments [13, 22] can be generated by suitable voltages applied to the side gates. The schematic of the considered nanostructure is presented in Fig. 16(a), together with an exemplary material profile of heterostructure in which
the two coupled quantum wells can be realized experimentally [Figs. 16(b)].

Figure 16: (a) Schematic of the bilayer nanowire with QPC. (b) Cross section of an exemplary realization of the bilayer nanowire based on Al$_{0.48}$In$_{0.52}$As/Ga$_{0.47}$In$_{0.53}$As double quantum wells. [H7].

In the paper, we used the two-band model with the Hamiltonian of the SO coupling induced by the lateral electric field given by

$$H_{SO} = \beta |e| F_y \hat{k}_x \mathbf{1} \otimes \sigma_z - \beta \delta |e| F_y \tau_y \otimes \sigma_x + \beta_{12} \tau_x \otimes \left( \sigma_x \hat{k}_y - \sigma_y \hat{k}_x \right), \quad (22)$$

where $\hat{k}_{x(y)} = -\partial/\partial x(y)$ is the wave vector operator, $\beta$ is the lateral Rashba SO coupling constant, $\beta_{12}$ is the inter-subband lateral Rashba SO coupling constant, $\delta = \langle 1, \sigma |\partial/\partial z| 2, \sigma \rangle$ is the inter-subband coupling constant and $(\sigma_x, \sigma_y, \sigma_z)$ and $(\tau_x, \tau_y, \tau_z)$ are the Pauli matrices defined in the spin and orbital space, respectively. To calculate the spin-dependent conductance through the nanostructure we used the Landauer formula with the transmission coefficients determined numerically by the scattering matrix method.

Figure 17: (a) Spin-dependent conductance $G^\uparrow$, $G^\downarrow$ as a function of the Fermi energy $E_F$ for different values of $\delta$. (b) Spin polarization of current $P$ as a function of the Fermi energy $E_F$ for $\delta = 10^{-2}$ nm$^{-1}$. [H7].

Fig. 17(a) displays the conductance $G^{\uparrow\downarrow}$ as a function of the Fermi energy $E_F$ for two values of the inter-subband coupling constant $\delta$ with $\beta_{12} = 0$. Red and blue curves correspond to $G^{\uparrow}$ and $G^{\downarrow}$, respectively. For $\delta = 0$, depicted by the black dashed curve, $G^{\uparrow} = G^{\downarrow}$ and the two conductance steps are due to the subsequent subbands passing through the Fermi level in the QPC region. For the nonzero inter-subband coupling, i.e. for $\delta = 10^{-2}$ nm$^{-1}$, the conductances
$G^\uparrow$ and $G^\downarrow$ differ from each other in some range of the Fermi energy, which leads to the almost full spin polarization of current $[P = (G^\uparrow - G^\downarrow)/(G^\uparrow + G^\downarrow)]$ as presented in Fig. 17(b).

The proposed spin filtering effect results from the inter-subband coupling induced by the transverse electric field and the strong confinement in the QPC region. It was explained in detail in the paper [H7] on the basis of the charge and spin density distributions. Here we will describe it based on the schematic presented in Fig. 19 and the spin density distributions shown in Fig. 18, where $s_{n\sigma}^m$ is the $z$-component of the spin density in the band $m$ for the case when the electron with spin $\sigma$ is injected into the system in the band $n$.

![Figure 18](image1.png)

**Figure 18:** The $z$-component of the partial spin density distributions $s_{n\sigma}^m$, where $m$ is the index of the subband in which the spin density distribution is presented, while $n\sigma$ denote the index of the subband, including spin, from which the electrons are injected into the nanowire. [H7].

![Figure 19](image2.png)

**Figure 19:** Dispersion relation $E(k)$ in the lead IN and OUT as well as in the nanowire with the SO interaction and in the QPC region. The dashed horizontal line denotes the Fermi energy for which the spin density distribution maps are presented in Fig. 18. (b) Schematic illustration of the possible transmission processes through the nanowire. [H7].

Even a cursory analysis of Fig. 18 indicates that the spin polarization of current at the output corresponds to the transport of electrons injected in the states $|1, \uparrow\rangle$ and $|2, \uparrow\rangle$. To explain this let us consider the transport processes for electrons injected into the nanowire from the subsequent subbands (i) $|1, \uparrow\rangle$, (ii) $|1, \downarrow\rangle$, (iii) $|2, \uparrow\rangle$ and (iv) $|2, \downarrow\rangle$.

(i) For the given Fermi energy, the electron injected into the nanowire in the state $|1, \uparrow\rangle$ flows through the system conserving its spin - see Fig. 18 (in fact, it changes slightly in the SO region). Since the Fermi energy is higher than the bottom of the state $|1, \uparrow\rangle$ in the QPC region [Fig. 19(a)], the electron is transmitted through QPC giving a positive contribution to the spin-up polarized current at the output.
The spin dynamics of the electron injected in the state $|1, \downarrow\rangle$ is much more interesting due to the hybridization of states $|1, \downarrow\rangle$ and $|2, \uparrow\rangle$ induced by the SO interaction - in the SO region we can see the repulsion of states $|1, \downarrow\rangle$ and $|2, \uparrow\rangle$ (anticrossing). Due to the hybridization, the electron initially injected in the state $|1, \downarrow\rangle$, in the region of the SO coupling, is transmitted to the state $|2, \uparrow\rangle$ as a result of the Landau-Zener transition. Since the bottom of the band $|2, \uparrow\rangle$ in QPC is higher than the Fermi energy, the electron is backscattered from QPC and does not contribute to the electron current at the output.

(iii) The opposite process occurs for the electron injected in the state $|2, \uparrow\rangle$. Due to the hybridization of states $|1, \downarrow\rangle$, $|2, \uparrow\rangle$, the electron initially injected in the state $|2, \uparrow\rangle$ is transmitted to the state $|1, \downarrow\rangle$ before reaching the QPC region. Since the bottom of the state $|1, \downarrow\rangle$ in QPC is below the Fermi energy, the electron flows through QPC and just after passing through QPC, it is transmitted to $|2, \uparrow\rangle$ again, giving contribution to the spin-up polarized current at the output - see Fig. 18.

(iv) For a given Fermi energy, the electron injected in the state $|2, \downarrow\rangle$ reaches QPC almost without changing its spin state. Since the Fermi energy is lower than the bottom of the state $|2, \downarrow\rangle$ in QPC [Fig. 19(a)], the electron is backscattered.

Summing up, the spin polarization of current at the output is mainly related to the transport processes (i) and (iii). All processes described above are schematically presented in Fig. 19(b).

In the paper [H7], the proposed spin filtering effect was also analyzed as a function of the inter-subband SO interaction $\beta_{12}$ and the inter-subband coupling $\delta$. We showed that the spin filtering effect occurs only in a certain range of $\delta$ and the spin polarization of current oscillates in $\beta_{12}$ gradually decreasing its amplitude.


The paper [H8] is related to my current scientific activity which mainly concerns topological states of matter, in particular, Majorana states realized in semiconductor/superconductor hybrid structures [39, 40, 41]. The non-abelian statistics of Majorana fermions makes them interesting for quantum computations [50]. Since even an elementary braiding operation of Majoranas requires at least the three-terminal junction, the current interests of experimentalists are directed towards the multi-terminal semiconductor/superconductor hybrid structures [51, 52, 53] as prototypes for topological quantum gates. In such structures, the magnetic field perpendicular to the substrate (in contrast to the experiments on single nanowires, where the magnetic field is directed along the nanowire axis) becomes more preferable as it allows to induce the Majorana states in all the branches - for each of them the magnetic field is perpendicular to the effective Rashba field. However, for the perpendicular alignment of the magnetic field, the orbital effects start to play an important role. Their influence on the Majorana state decay length is the subject of the paper [H8] where we showed that the orbital effects from the magnetic field not only do not destroy the Majorana states, as commonly thought, but even reduce their decay length being preferable for topological quantum computing.

In our paper [H8] we considered a semiconductor 2D nanowire with a strong spin-orbit interaction (InSb) subjected to the magnetic field perpendicular to the nanowire axis. In the weak coupling regime where the semiconductor/superconductor interface is non-transparent, we can assume a uniform superconducting gap $\Delta$ in the whole wire. Hamiltonian of the system is given by

$$H = \left(\hbar^2k^2/2m^* - \mu\right)\sigma_0\tau_z + \Delta\sigma_0\tau_x + \alpha(\sigma_xk_y - \sigma_yk_x)\tau_z + E_z\sigma_z\tau_0,$$

where $m^*$ is the effective mass, $\mu$ is the chemical potential, $\alpha$ is the spin-orbit coupling constant, $E_z = g\mu_B B$ is the energy of Zeeman splitting, $\sigma_i(\tau_i)$ are Pauli matrices acting on spin-
particle-hole degrees of freedom, respectively, and \( \mathbf{k} = -i\nabla + e\mathbf{A}/\hbar \cdot \mathbf{\tau}_z \) is the canonical momentum. We assume the vector potential in the Lorentz gauge, \( \mathbf{A} = [-yB, 0, 0] \).

In our paper, we derived an analytic formula for the Majorana state decay length which takes into account the orbital effects of magnetic field. For this purpose, the Hamiltonian (23) was expressed in the basis of two lowest eigenstates of the infinite quantum well of width \( W \). The two-band model seems to be justified since, as shown by the experiments, the most convenient situation for realization of Majoranas is when a single band of transverse quantization is occupied. The perpendicular magnetic field, however, couples the transverse states due to the paramagnetic term (\( \sim k_x yB \)) which comes from both the kinetic energy and the spin-orbit interaction. For simplicity, in our model we consider only the first excited state whose the coupling with the ground state is the largest. Hamiltonian in the two-band model takes the form

\[
H = \begin{pmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{pmatrix},
\]

with

\[
H_{11(22)} = H_{1D} + (E_{1(2)} + E_{\text{dia}})\sigma_0 \tau_z,
\]

\[
H_{12} = H_{21} = \varepsilon_p k_x \sigma_0 \tau_0 + E_p^{\text{SO}} \sigma_y \tau_0 + E_\perp^{\text{SO}} \sigma_x \tau_z,
\]

where \( E_n = n^2 \pi^2 \hbar^2 / 2mW^2 \) is the energy of orbital excitation in the \( y \)-direction, \( E_n^{\text{dia}} = \langle \Psi_n | y^2 | \Psi_n \rangle \hbar^2 B^2 / 2m \) is the diamagnetic term in the \( n \)-th band [\( \langle \Psi_1 | y^2 | \Psi_1 \rangle = (\pi^2 - 6)W^2 / 12\pi^2 \) and \( \langle \Psi_2 | y^2 | \Psi_2 \rangle = (2\pi^2 - 3)W^2 / 24\pi^2 \)], \( \varepsilon_p = -\langle \Psi_1 | y | \Psi_2 \rangle eB / m = 16W eB / 9m\pi^2 \) is the paramagnetic energy coming from the kinetic term, \( E_p^{\text{SO}} = \langle \Psi_1 | y | \Psi_2 \rangle \alpha eB / \hbar = -16W eB / 9\hbar\pi^2 \) is the paramagnetic term originating from the spin-orbit interaction and \( E_\perp^{\text{SO}} = i\alpha \langle \Psi_1 | \partial / \partial y | \Psi_2 \rangle = 8i\alpha / 3W \) is the correction to the transversal energy resulting from the spin-orbit interaction. In equation (25)

\[
H_{1D} = \left( \frac{\hbar^2 k_x^2}{2m^*} - \frac{\mu}{m^*} \right) \sigma_0 \tau_z + \Delta \sigma_0 \tau_x - \alpha k_x \sigma_y \tau_z + E_\perp \sigma_z \tau_0.
\]

If we assume that \( E_2 \) is the largest energy in the system, the two-subband model (24) can be reduced to the effective Hamiltonian for the ground state with renormalized parameters resulting from the coupling between the bands induced by the orbital effects. Using the folding-down transformation

\[
\mathcal{H}(E) = H_{11} - H_{12}(H_{22} - E)^{-1} H_{21},
\]

we obtain Hamiltonian similar to (27) with the renormalized effective mass \( \tilde{m}^* \), chemical potential \( \tilde{\mu} \) and Rashba spin-orbit constant \( \tilde{\alpha} \) given by the formulas

\[
\frac{1}{\tilde{m}^*} = \frac{1}{m^*} - \frac{2\varepsilon_p^2}{\hbar^2 E_2},
\]

\[
\tilde{\mu} = \mu - E_1 - E_1^{\text{dia}} + \frac{(E_p^{\text{SO}} - E_\perp^{\text{SO}})^2}{E_2},
\]

\[
\tilde{\alpha} = \alpha + 2 \frac{E_p^{\text{SO}} \varepsilon_p}{E_2}.
\]

The Majorana state decay length was derived analytically based on the model presented in Ref. [73] and is given by

\[
\xi \simeq \frac{1}{\tilde{\alpha} \Delta} \sqrt{\left( \frac{\hbar^2}{\tilde{m}^*} \tilde{\mu} + \tilde{\alpha}^2 \right)^2 + \left( \frac{\hbar^2}{\tilde{m}^*} \right)^2 \left( E_2^2 - \Delta^2 - \tilde{\mu}^2 \right)}.
\]
Fig. 20 presents the Majorana state decay length $\xi$ in the topological regime, as a function of the magnetic field $B$ for two values of the Rashba coupling constant, $\alpha$. We see that the inclusion of the orbital effects in the kinetic term (curve dotted with dots) leads to a significant reduction in the decay length $\xi$ in respect to the situation in which the orbital effects are not taken into account (dotted curve). However, the inclusion of the orbital effects by the kinetic term and the spin-orbit Hamiltonian (a solid curve with crosses) weakens the observed effect. Note, however, that in this case $\xi$ is still lower than that obtained in the absence of the orbital effects. In the paper [H8] we analyzed in detail the impact of individual terms on the Majorana state decay length. In addition, our numerical calculations of decay length taking into account the coupling with the higher transverse states showed that the observed reduction of $\xi$ is much stronger than it results from the presented two-band model.


The paper [H9] is an extension of the study presented in Ref. [H8] to the strong coupling at semiconductor/superconductor interface. It is particularly important in the context of the recent experiments with a thin superconducting shell Al epitaxially deposited on the facets of semiconductor nanowire [40, 53]. In the paper [H9] we studied the impact of the orbital effects on the induced superconducting gap. We showed that the correct way of taking into account the orbital effects requires minimizing the free energy of the system which guarantees the stationarity. It turns out that the previous theoretical papers [74, 75] on the orbital effects showed that a slight tilting of the magnetic field with respect to the nanowire axis results in the closing of the effective superconducting gap, and thus destroys the topological phase. Nevertheless, as we demonstrated, this conclusion is a result of an incorrect inclusion of the orbital effects - the authors used the vector potential which does not guarantee the stationarity, i.e. $j_c \neq 0$.

In the first part of the paper [H9] we considered a 2D Majorana nanowire with a magnetic field perpendicular to the nanowire axis, described by the Lorentz gauge $\mathbf{A} = [-yB, 0, 0]$. The vector potential $\mathbf{A}$ was introduced into our numerical model by the appropriate Peierls substitution

$$t_{n,m} \rightarrow t_{n,m} \exp \left(-\frac{e}{\hbar} \int \mathbf{A} \cdot d\mathbf{l}\right).$$

For the nanowire located symmetrically with respect to the $x$ axis, the value of the topological gap $\Delta^*$ as a function of the magnetic field $B$ and the chemical potential $\mu$ is presented in Fig. 21(a). $\Delta^*$ was determined based on the dispersion relations $E(k)$, as shown in the panel (b). Now, with the fixed gauge $\mathbf{A} = [-yB, 0, 0]$, let us shift the nanowire so that it would be placed asymmetrically with respect to the $x$ axis. As we
Figure 21: (a)(c) Topological energy gap $\Delta^*$ as a function of the magnetic field $B$ and the chemical potential $\mu$. (b)(d) Dispersion relations $E(k)$ calculated for $\mu = 2.3\, \text{meV}$ and $B = 0.4\, \text{T}$. Top panels: results for the nanowire located symmetrically with respect to the $x$-axis. Bottom panels: results for the nanowire located asymmetrically with respect to the $x$-axis. [H9].

We can see [Fig. 21(d)] the dispersion relations $E(k)$ tilt due to the orbital effects and the topological gap closes for a small magnetic field $B$. Fig. 21(c) shows that the range of parameters for which $\Delta^* \neq 0$ is significantly reduced. Such calculations lead to the striking conclusion that the orbital effects of the magnetic field close the topological gap in the parameter range where the gap is still open [compare Fig. 21 (a) and (c)]. Let us emphasize that the above thought experiment has nothing to do with violating of the gauge invariance, i.e., the proper gauge transformation does not change the results, but also it does not answer the question which of the maps (a) or (c) is physically correct. It turns out that the results that we obtain after translation of the system results from the non-zero superconducting current $j_c$ which is generated in the system by the vector potential. Therefore, the results presented on the panels (c) and (d) describe the excited state with $j_c \neq 0$. As shown in our paper [H9], the correct inclusion of the orbital effects in the stationary state requires $j_c = 0$. This condition can be guaranteed by minimizing the free energy which for $T = 0$ reduces to the condensation energy defined as the difference of energy in the superconducting and normal state. Assuming the gauge in the form $A = [- (y - y_0)B, 0, 0]$ we calculated the condensation energy as a function of the parameter $y_0$ for the nanowire located asymmetrically around the $x$-axis from Fig. 21(c).

The distinct minimum of the condensation energy which ensures the stationarity of the system ($j_s = 0$) is localized exactly in the middle of the nanowire (Fig. 22). The map of the topological gap calculated for the vector potential determined in this manner, is exactly the same as presented in Fig. 21(a). For a simple translation of a homogeneous nanowire, the form of a vector potential can be easily guessed. However, other forms of symmetry breaking, e.g. by applying the transverse electric field (presented in the paper [H9]), requires determination of the vector potential by minimizing the free energy. From the experimental point of view, the most important systems with a broken symmetry is a semiconductor nanowire with a thin superconducting shell deposited on the facets.

In the paper [H9], we developed a numerical model to determine the induced superconducting gap in the strong semiconductor/superconductor coupling regime, taking into account the orbital effects and the substantial difference in the effective mass and the chemical potential between superconductor and semiconductor. By the appropriate choice of the gate potential and the
Figure 22: Condensation energy $\varepsilon_\Delta$ as a function of the parameter $y_0$ calculated for the nanowire located asymmetrically around $x$-axis, presented in Fig. 21(c). [H9].

Figure 23: Condensation energy $\varepsilon_\Delta$ as a function of $y_0$ calculated for the semiconductor/superconductor hybrid nanostructure presented in the inset. [H9].

Figure 24: Dispersion relations $E(k)$ of the semiconductor/superconductor hybrid nanostructure calculated for the magnetic field (a) $B = 0.2$ T and (b) $B = 0.4$ T. [H9].

band offset between the materials, the system under consideration (Fig. 23) was set so that the induced gap in the semiconductor was close to the energy gap in the superconducting shell. As shown in Ref. [H9], it is only possible if the lowest electron state in semiconductor nanowire is strongly coupled with one of the superconducting states. For such a system, we studied the orbital effects on the induced superconducting gap. In order to take into account the orbital effects that provides the stationary ($j_c = 0$), we minimized the condensation energy assuming the gauge in the form $A = -((y - y_0)B, 0, 0)$. As presented in Fig. 23, the minimum of the energy $\varepsilon_\Delta$ corresponds to $y_0$ located exactly in the middle of the superconductor. For the gauge determined in this manner the dispersion relations tilts as presented in Fig. 24. The critical field obtained from the numerical calculations, $B_c = 0.4$ T, at which the gap closes (we assumed Al
thickness of about 10 nm which corresponds to the thickness used in the experiments), agrees with the critical field measured in the experiment [45].


In all the papers presented above, the value of the Rashba spin-orbit constant plays an important role. On the one hand, the modification of this parameter by means of external gate potentials allows to control the spin in spintronics nanodevices, on the other side, the Rashba constant determines the topological gap in semiconductor/superconductor nanostructures. For this reason, in the paper [H10], we studied the Rashba SO coupling constant in homogeneous semiconductor nanowires. The paper [H10] is a result of my current cooperation with a group of theoreticians from the University of Modena, in the framework of which we deal with the electronic structure calculations for semiconductor core-shell nanowires.

In the paper [H10] we used the 8-band $k \cdot p$ approximation adopted for semiconductor nanowires to determine the Rashba SO coupling constant for different gate configurations. In this model, the SO Hamiltonian for the conduction electrons takes the form

$$H_{SO} = (\alpha_x \sigma_x + \alpha_y \sigma_y)k_z,$$

with

$$\alpha_x(x, y) \approx \frac{1}{3} p^2 \left( \frac{1}{E_0^2} - \frac{1}{(E_0 + \Delta_0)^2} \right) \frac{\partial V(x, y)}{\partial y},$$

$$\alpha_y(x, y) \approx \frac{1}{3} p^2 \left( \frac{1}{E_0^2} - \frac{1}{(E_0 + \Delta_0)^2} \right) \frac{\partial V(x, y)}{\partial x},$$

where $E_0$ is the energy gap, $\Delta_0$ is the split-off band gap, $V(x, y)$ is the self-consistent potential being the sum of the Hartree potential and the potential generated by the gates, and $P$ is the conduction-to-valence band coupling.

The intra and inter-subband SO coupling constants are given by

$$\alpha_{nm}^{xy} = \int \int \psi_n(x, y)\alpha_{x(y)}(x, y)\psi_m(x, y)dxdy,$$

where $\psi_n(x, y)$ is the in-plane envelope function.

The calculations were carried out in two regimes: (i) at constant chemical potential, $\mu = const$, and (ii) at constant electron concentration, $n_e = const$, which correspond to two different realization of the experiment. In our theoretical model, the electron-electron interaction was taken into account in the mean field approximation within the Schrödinger-Poisson method. We considered zincblende nanowires grown along [111] for which Dresselhaus SO interaction is negligibly small due to the inversion symmetry.

I started from GaAs nanowires which is not a strong SOC material, however, it is the material of choice for transport experiments, due to its high mobility. Fig. 25 presents the Rashba SO coupling constants as a function of the gate potential for different gate configurations. Due to the inversion symmetry $\alpha_{11}^x = \alpha_{11}^y = 0$ for $V_g = 0$. As the gate voltage is switched on, $\alpha_{11}^i \neq 0$, with $i$ being the direction of the axis of symmetry broken by the field. So, for example, $\alpha_{x}^1 = 0$ in the panels (a) and (c), but not in the panel (b). On the other hand $\alpha_{x}^1 \neq 0$ in all configurations, since gates remove inversion symmetry about $x$ in all cases. The evolution of $\alpha_{x}^1(V_g)$ is strongly asymmetric which results from the complex interplay between quantum confinement from the NW interfaces, the gate-induced electric field and the self-consistent field due to electron-electron interaction. Their combined interaction was discussed in detail in our paper [H10] based on the self-consistent potential profiles and the electron density distributions.
Our calculations for the bottom gate configuration showed very interesting behavior of the Rashba constant around $V_g = 0$. Namely, for high electron concentrations, the SO coupling coefficient shoots up around $V_g = 0$, as presented in Fig. 26(a). Interestingly, the SO coupling susceptibility $\chi = d\alpha_{x,11}^{11}/dV_g|_{V_g=0}$ changes non-linearly as a function of the electron concentration [Fig. 26(b)]. This behavior corresponds to the strength of the Coulomb interaction. In the low electron concentration regime, the electron-electron interaction is negligible, quantum confinement from interfaces dominates, and at $V_g = 0$ the conduction band energy is nearly flat. As a results, the electron density and the envelope function of the ground state are localized in the center of the NW and exhibit a circular symmetry. The charge distribution is hardly modulated by the potential applied to the gate, and only slightly shifted upward or downward leading to the slight changes of the Rashba parameter. In the high concentration regime the electron-electron interaction dominates and total energy is minimized by reducing repulsive Coulomb energy, at the expense of localization energy. Accordingly, electrons move outwards and accumulate near the facets. At sufficiently high electron concentration charge localize in quasi-1D channels at the edges, a minor part of the charge sits at the facets, while the core of the wire is totally depleted. This symmetric edge localization is easily destroyed by any asymmetry introduced by the gate potential which leads to the shooting up of the Rashba constant around $V_g = 0$. 

In the second part of the paper [H10], the developed theoretical model was used to determine the Rashba constant for the experimental setup presented in Ref. [31], in which the measured...
value of the Rashba parameter for InSb nanowires was $\alpha = 50 – 100$ meVnm. The calculations were carried out for the same geometrical parameters as in the experiment - InSb nanowires with two gates, top and bottom, separated from the nanowire by dielectric layers with the specified thicknesses [Figs. 27(a)]. In Fig. 27(b)(c) we present the Rashba constant $\alpha_{x}^{11}$ as a function of the top and bottom gate voltage $V_{tg}$ and $V_{bg}$. The voltage ranges under consideration correspond to these used in the experiment [Fig. 27(b)]. Note that the Rashba constant can be controlled by $V_{bg}$ in the narrow range which results from the strong screening of the electric field coming from this gate by the thick dielectric layer SiO$_2$ ($d_{SiO_2} = 285$ nm). For $V_{bg} = 0$, the Rashba SO constant $\alpha_{x}^{11} \neq 0$, which results from the asymmetry introduced into the system by the asymmetrical location of the gates. Much better control of the Rashba constant is obtained by the top gate, strongly coupled to the nanowire by a thin dielectric layer. As shown in Fig. 27(c), by applying a positive voltage to the top gate we can change the Rashba constant $\alpha_{x}^{11}$ in a wide range. Note that for voltages used in the experiment, we get the value of the Rashba constant corresponding to the experimentally measured.

In the paper [H10], besides the intra-subband SO constants, the inter-subband SO constants were also analyzed. We showed that for certain experimental geometries and high electron concentrations, the inter-subband Rashba coupling constants reach values close to the intra-subband constant.


My research on the spin-orbit interaction in semiconductor nanowires has been recently extended to core-shell wires. In the paper [H11] we showed that in InAs/InAsP nanowires, the interfacial SO component ($\alpha_{int}^R$) related to the occurrence of the interface between the core and the outer shell, may be greater than the electrostatic component ($\alpha_R^E$) resulting from the electric
field generated by the gates [Fig. 28(a)]. As shown in Fig. 28(b), the ratio $\alpha_{int}/\alpha_V$ depends on the alloy composition of the shell, which determines the height of the potential barrier between the core and the shell. For a certain range of $x$, the potential barrier is small enough that the electron penetrates into the shell leading to the enhancement of the Rashba constant, related to a strong electric field at the interface of both materials.

Figure 28: (a) Lines: total, electrostatic and interfacial SOC constants vs gate voltage $V_g$, according to labels. Dots: total SO constant for an equivalent homogeneous InAs NW. Inset: ratio between interfacial and electrostatic components, $\alpha_{int}/\alpha_V$ as a function of $V_g$. (b) The interfacial (blue circles) and electrostatic (red circles) SOC constants vs InAs$_{1-x}$P$_x$ alloy composition, $x$. Inset: $\alpha_{int}/\alpha_V$ vs $x$. [H11]

The enhancement of the SO coupling in core-shell wires was observed experimentally in Ref. [33], and our finding is one of the possible theoretical explanation of the observed effect.

4.3 Summary

The series of papers [H1]–[H11] presents the results of comprehensive study on the role of spin-orbit interaction and orbital effects in semiconducting and superconducting low-dimensional structures, both in the context of their applications in spintronics, and their role in the formation of Majorana states in semiconductor/superconductor hybrid systems. The most important results of my study are as follows:

1. I proposed two models of spin filters based on semiconductor materials: (i) in the Y-shaped structure with QPC and (ii) in the bilayer nanowire with QPC,

2. I developed a theoretical model of the spin transistor based on semiconductor nanowires for the analysis of the experiment [25] and obtained good agreement with the experiment,

3. I investigated the transport properties of spin transistor based on the coupled quantum wells and showed that the inter-subband SO coupling reduces the ratio $G_{on}/G_{off}$,

4. I proposed the new model of spin transistor based on the helical field in which the transistor action is induced by the electric field. Moreover I explained the appearance of the additional dips in conductance that appear in the non-adiabatic regime,

5. I demonstrated that the FF phase can be induced by the orbital effects in superconducting nanowires (in contrast to the common knowledge that the orbital effects are detrimental to the FF phase),

6. we showed that the orbital effects reduces the Majorana state decay length,

7. I developed a theoretical model to calculate the Rashba SO coupling constants in semiconductor nanowires for the analysis of the experiment [45] and obtained good agreement with the experiment,
8. I demonstrated the enhancement of Rashba SO coupling in core-shell nanowires induced by the interfacial effect.

In my opinion, results of my studies contribute to better understanding of the role of spin-orbit interaction and orbital effects both in the spin control and the topological states of matter and can be used to build new spintronics devices or topological quantum gates based on semiconductor/superconductor hybrid nanostructures.

4.4 References


5 Other scientific achievements

5.1 Before obtaining the PhD degree

During my PhD study, under the supervision of prof. Janusz Adamowski and in cooperation with dr hab. Bartłomiej Spisak and dr hab. Maciej Wołoszyn from the Faculty of Physics and Applied Computer Science AGH, I dealt with the simulations of the electron transport in resonant tunneling nanostructures. The used theoretical model was based on the Wigner function for description of quantum systems in the phase space. I developed the time-independent and time-dependent Wigner-Poisson method for the simulations of electron transport in the layered semiconductor heterostructures [P1].

In this respect, my study starts from the analysis of non-linear effects, i.e. bistability and high frequency current oscillations in the triple-barrier resonant tunneling diode [P2, P3]. We showed that in such systems there are two types of bistability related to the accumulation of charge in the main quantum well and in the emitter region [P2]. In the paper [P3] we described spontaneous high-frequency (THz) current oscillations which, as we showed, result from the coupling between quasi-bound states formed in the nanodevice. The ability to control the oscillations frequency by the width of the barrier separating both the wells was presented in Ref. [P4].

A considerable part of my PhD studies concerned the application of resonant-tunneling heterostructures as effective spin filters based on semiconductor materials. If a quantum well in the resonance-tunneling diode is embedded in the dilute magnetic semiconductor, the giant Zeeman splitting of quasi-bound states in the quantum well leads to the situation that the resonant tunneling conditions for spin-up and spin-down electrons are met for different bias voltages. This leads to the spin separation of the resonant current and consequently the spin polarization of current which can be controlled by the external electric field. The splitting of the resonant current peak into two peaks corresponding to the different spin components, was experimentally reported by Slobodskyy et al. [Phys. Rev. Lett. 90, 246601] in the paramagnetic resonant tunneling diode based on ZnSe/ZnBeSe/ZnMnSe. However, to fully describe the spin filtering effect in the magnetic resonant-tunneling structures we should take into account the non-linear phenomena that may appear in these nanodevices. Non-linear transport phenomena and their influence on the spin polarization of current in the paramagnetic resonant-tunnel diodes were the subject of the papers [P5, P6, P7].

In Ref. [P6] we demonstrated that in the paramagnetic resonant tunneling diodes, the spin polarization of current calculated with increasing bias voltage is different from that obtained with decreasing bias voltage - bistability of the spin polarization. This phenomenon was explained as resulting from the electrostatic interaction between electrons accumulated in the emitter and in the main quantum well. Our further study on the spin polarization of current in paramagnetic resonant tunneling structures concerned the behavior in time. Time-dependent transport simulations showed that in the negative differential resistance region, the spin polarization of current is not constant but oscillates with a high (THz) frequency [P7]. This phenomenon was explained in Ref. [P7] where we demonstrated that the high frequency oscillations of current result from the coupling between quasi-bound states in the emitter and the main quantum well. The culmination of our study on the spin filtering effect in magnetic resonant-tunneling structures was the paper [P8]. In Ref. [P8] we proposed the model of spin filter operating at room temperature. The proposed filter was based on the resonant-tunneling structure in which both the emitter layer and the main quantum well were embedded in the ferromagnetic semiconductor with the antiparallel magnetization. Our calculations showed that in the proposed structure, the spin polarization of current \( P \approx 35\% \) at room temperature which was the largest value of the spin polarization predicted theoretically at room temperature, at that time.

Moreover, during the six-month academic internship at University of Leeds in the group of prof. B. Hickey I dealt with the simulations of electron transport in ferromagnet/superconductor
junctions for the analysis of the experimental data from the PCAR method (Point Contact Andreev Reflection) [P9].


5.2 After obtaining the PhD degree

My scientific activity after obtaining the PhD degree oscillated around two topics. The first one is a natural continuation of the research undertaken during the PhD study and concerns a spin dependent electron transport in semiconductor nanoscopic structures. The second, which is the fruit of my scientific internship at University of Antwerp in the group of prof. F. Peteers, concerns superconductivity in the nanoscale regime. My current studies are also focused on the properties of semiconductor/superconductor hybrid nanostructures as the combination of both the mentioned topics.

5.2.1 Electronic transport in semiconductor nanostructures

Since the recent experiments show that resonant tunneling structures can be electrostatically generated in nanowires (by the use of appropriate electrodes applied to the wire), further studies of our group focused on the spin-dependent electron transport in semiconductor nanowires. Our research in this area received funding under the OPUS grant funded by the National Science Center which I was one of the main executor. Results of our preliminary calculations [P1] showed that the resonant current oscillates as a function of the emitter length. In Ref. [P1] this phenomenon was explained as resulting from the Stark resonances which appear after applying the electric field along the nanowire axis. Our further studies demonstrated that the Stark
resonances in nanowires can lead to a significant increase of the magnetoresistance in the external magnetic field [P2]. Moreover, in Ref. [P3] we found a change of the magnetoresistance sign induced by the strong transverse confinement.

If the first and supreme condition for spintronics development is the creation of the spin polarized current, the second necessary condition is related to effective methods of the spin control in semiconductor materials. In our paper [P4] we studied the spin dynamics under the spin-orbit interaction in a two-dimensional, cylindrical electron gas in core-shell nanowires. The spin-orbit effect originating from the radial electric field and the electric field directed along the nanowire axis were considered separately. We demonstrated that in the case of the radial electric field, the superposition of states with the same orbital quantum number leads to the spin precession around the nanowire axis. On the other hand, the Rashba SO interaction coming from the electric field applied along the nanowire axis leads to the oscillations of the $z$-component of spin with the pronounced beating effect.

The study on the electronic properties of core-shell nanowires are the current subject of my interests. For this purpose, I established cooperation with theoreticians from University of Modena who have experience in the electronic structure calculations for semiconductor nanostructures. The result of my cooperation with this group are two papers which are the subject of the scientific achievement [H10, H11].

5.2.2 Superconductivity in the nanoscale regime

The huge development of the epitaxial growth techniques that took place in the last decade makes superconductivity in the nanoscale regime, along with the high-temperature superconductors, one of the most interesting research object in the field of superconductivity. These studies are motivated by the desire to answer fundamental questions about the size of the system below which superconductivity disappears, and the influence of the confinement on superconducting properties. The experiments carried out in this field have led to the observation of many fascinating and unexpected phenomena that in many cases are still awaiting for the theoretical explanation. One of the most spectacular, just three years ago, is the increase of the critical temperature for the FeSe monolayer on SrTiO$_3$ up to 100 K ($T_c$ for bulk FeSe is 8 K) [Nat. Mater.,14, 285 (2015)]. Undoubtedly, the reduction of size to nanometers results in significant changes in the electronic structure, phonon dispersion relations and electron-phonon interaction that involves the changes in superconducting properties of the considered nanostructures. Measurements for thin Pb layers deposited on Si (111) showed that the critical temperature oscillates as a function of the layer thickness with a period of two atomic layers [Science 306, 1915 (2004), Nat. Phys, 6, 104 (2010)]. My research in this field received funding under the JUVENTUS PLUS grant funded by the Ministry of Science and Higher Education in years 2013-2015. Our studies carried out for thin metallic layers allowed to obtain experimentally observed effect of the critical temperature oscillations and agree quantitatively with the experiment [P5, P6].

The recent study on superconductivity in low-dimensional structures concerns the superconductor - to - normal metal transition induced by the external magnetic field. Experimentally, the oscillations of the critical field as a function of the layer thickness were observed in [Phys. Rev. Lett 111, 057005 (2013)] and, as explained in Ref. [P6], this phenomenon is induced by the quantum size effect. Nevertheless, the most interesting seems to be the fact that the measured in-plane critical field, when the orbital effects are strongly reduced by the confinement in the growth direction, is much greater than the paramagnetic critical field determined from the Clogstone-Chandrasekhar (CC) formula. The fact that the CC formula is not met in thin superconducting layers was partially explained in our recent paper [P7]. Note that the CC formula was derived with the assumption of spatially homogeneous energy gap. This condition is not met in superconducting nanostructures in which the energy gap distribution is not homogeneous in the confinement direction. In the paper [P7] we proposed the phenomenological formula for the critical field in the Pauli limit in superconducting thin layers with a non-uniform energy gap. The influence of orbital effects on the critical magnetic field was analyzed in the paper [P8].
The explanation proposed in Ref. [P7] is not the one and only mechanism that can be responsible for the increase of the critical field above the paramagnetic CC limit. The in-plane magnetic field may be a source of an unconventional electron pairing with non-zero momentum of Cooper pairs \((k_\uparrow, -k + q \downarrow)\). This superconducting phase is called the FFLO phase. According to the original idea, the FFLO phase arises as a result of the Zeeman spin splitting in the magnetic field. The mismatch of the Fermi vector between the bands with opposite spins can be compensated by the non-zero momentum of Cooper pairs - pairing \((k_\uparrow, -k + q \downarrow)\). Thus, destruction of the superconducting state related to the paramagnetic effect can be partially compensated by the FFLO phase formation which leads to the increase of the critical field. My research in this field [P9] showed that in metallic nanofilms the FFLO phase stability region oscillates with the nanofilm thickness in antiphase to the critical magnetic field oscillations. Moreover, the FFLO phase stability region split into sub-phases, the number of which corresponds to the number of subbands participating in the electron pairing. The in-depth analysis of the FFLO phase stability conditions induced by the Zeeman effect and the orbital effects was carried out in Ref. [P10], in which the superconducting wire with the core-shell structure was considered. As shown, by changing geometrical parameters we can control the contribution of the orbital effects, and thus switch the system from the regime in which it exhibits characteristics of nanowires, to the regime with properties observed only for nanofilms. In the paper [P10], for both the regimes we analyzed the conditions of the FFLO phase stability in terms of the orbital effects.

Unconventional superconductivity in the nanoscale regime is the current topic of my interests, which I realize with the group of theoreticians from the Academic Center of Materials and Nanotechnology, AGH. My research in this field is financed by the National Science Center under the SONATA grant, of which I am the leader (2017-2020).

5.2.3 Transport in semiconductor(normal metal)/superconductor hybrid system

In my research on spintronics, I focused my attention on hybrid nanostructures such as ferromagnet/ superconductor (FM/SC) and semiconductor/ superconductor (SEM/SC) junctions. Recent studies showed that such systems are promising for storing quantum information and implementation of quantum logic gates. Beside the Majorana states in SEM/SC hybrid systems, my recent scientific interests include the FM/SC junctions in term of the normal and anomalous Andreev reflections. I became interested in this topic during my scientific internship at the University of Leeds, where I collaborated with a group of experimentalists on the theoretical interpretation of measurements from the PCAR technique. In one of my last paper I studied the conductance through the ferromagnet/ conical-ferromagnet/ superconductor (FM/CM/SC) junction [P11]. In such systems, the anomalous Andreev reflections occur, when the electron incident at the superconductor interface is reflected as a hole with the same spin. As a result, the triplet pairing is induced in the s-wave superconductor. In our paper, using the Bogoliubov-de Gennes equations and the Blonder-Tinkham-Klapwijk formalism, I studied the impact of the adiabacity of transport through the CM layer on the anomalous Andreev reflections. We showed that in the non-adiabatic regime, the conductance reaches the maximum for the thickness of CM layers, for which an increase of the critical current was observed in the experiment. In this sense, the results of my calculations agree qualitatively with the experiment.

Our last paper concerns the electron penetration depth in a superconductor. In the paper [P12] we proposed a method for determining the length in which the electron penetrates into the superconducting region before it is reflected as a hole (Andreev reflection). Our method is based on the measurement of the Aharonov-Bohm oscillations period in SEM/SC hybrid nanostructures, in the form of the quantum ring or the hashtag structure.


